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Active Learning about Climate Change

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Abstract: We develop a climate-economy model with active learning. We consider three ways of active learning: improved observations, adding observations from the past and improved theory from climate research. From the model, we find that the decision maker invests a significant amount of money in climate research. Expenditures to increase the rate of learning are far greater than the current level of expenditure on climate research, as it helps in taking improved decisions. The optimal carbon tax for the active learning model is nontrivially lower than that for the uncertainty model and the passive learning model.

JEL classification: Q54

Key words: Climate policy; deep uncertainty; active learning; Bayesian statistical decision; integrated assessment; dynamic programming

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Active Learning about Climate Change

1 Introduction

It is well-known that deep – or fat-tailed – uncertainty substantially increases the stringency of climate policy (Tol, 2003; Weitzman, 2009).¹ This is summarized and emphasized as the Dismal Theorem by Martin Weitzman (2009). Most literature dealing with deep uncertainty about climate change focuses on its effect on welfare and policy – so-called tail effects – and its applicability in the Expected Utility framework. For more discussions on the Dismal Theorem see, among others, Hennlock (2009), Nordhaus (2011), Pindyck (2011; 2012), Weitzman (2011), Anthoff and Tol (2013), Millner (2013), and Hwang et al. (2013b). Seen from a different perspective, such impact of deep uncertainty implies that we can reap substantial benefits from reducing uncertainty – or learning.² This is because learning is faster in the tail, thus thinning the tail of climate impacts. Hwang et al. (2013a) show this hypothesis with their *passive* learning model. In the current paper we investigate this hypothesis with an *active* learning model.³

¹ By fat tails we mean that the probability density falls less than exponentially in the tail. See Nordhaus (2011) and Weitzman (2013) for the meaning of fat tails.

² Note that we follow the tradition of Bayesian statistical decision theory which requires that uncertainty or partial ignorance about a parameter can be summarized by a probability distribution (DeGroot, 1970). The distribution of an uncertain variable may be subjective, but it can be based on the support of physical theory (see Section 3).

³ In the stochastic control literature, active learning – also known as dual control, optimal experimentation, or optimal probing, as opposed to passive learning refers to the case in which a decision maker actively affects the rate of learning. See Kendrick (2005) for a review on the stochastic control literature.

We develop a climate-economy model with active learning. Although active learning has been dealt with in areas such as macroeconomics, this is a new addition to the literature on climate change.⁴ For instance, the seminal works on learning about climate change by Manne and Richels (1992), Kolstad (1996a,b), and Ulph and Ulph (1997) incorporate exogenous learning. In their model, information is exogenously given at some points in time, and thus learning has nothing to do with the action of the decision maker. More recent works including Kelly and Kolstad (1999), Leach (2007), Webster et al. (2008), Kelly and Tan (2012), Hwang et al. (2013a) consider endogenous (Bayesian) learning. In these papers a measure of (partial) ignorance about the true state of the world is introduced and its magnitude gets smaller as the decision maker gathers relevant information. Learning in this case is passive, however, since the decision maker does not have any explicit option to increase the speed of learning.

The implementation of active learning in an integrated assessment model of climate change is worthwhile for the following reasons. First, learning in climate science is active in the sense that a decision maker – or a researcher on her behalf – makes an explicit effort to gather information on uncertain variables. For instance, research on the climate process using a climate model or paleoclimate data enables us to formulate a probability distribution about the equilibrium climate sensitivity (Solomon et al., 2007).⁵ The decision maker makes an

⁴ The relevant literature for active learning model is as follows: for monetary policy (e.g., Wieland 2000; Bullard and Mitra, 2002; Yetman, 2003; Ferrero, 2007), for optimal consumer behaviors (e.g., Grossman et al., 1977), for technological innovations (e.g., Johnson, 2007), for economic growth (e.g., Bertocchi and Spagat, 1998), and for environmental policy (e.g., McKittrick, 2010; Van Wijnbergen and Willems, 2012).

⁵ The equilibrium climate sensitivity, which is one of the main parameters of interest in climate science, refers to the equilibrium global warming in response to a doubling of the atmospheric concentration of carbon dioxide, the major anthropogenic greenhouse gas.

explicit effort to promote such kind of research activities (e.g., R&D investment). The rate of learning is thus dependent upon such efforts.

Second, an active learning model gives a useful perspective on the relationship between climate and the economy. In a passive learning model, knowledge accumulates as time progresses and more observations become available. The action taken by the decision maker (such as emission reduction) also produces information. More emissions are justifiable if the gain from more emissions is higher than the extra damage of climate change plus the induced cost of learning. On top of this updating procedure, an active learning model takes a further role of policy decisions into account: increasing the rate of learning. In this framework, gains from learning consist of improved decisions. The costs of learning are the price of information such as observational costs and research investment. Balancing the gains and losses, the rate of learning is determined.

Previous papers on decision making under uncertainty and learning assume that knowledge grows by one observation per year. Instead, we consider three alternative ways of active learning: 1) improved observations (abbreviated IO), 2) adding observations from the past (abbreviated AO), and 3) improved theory from climate research (abbreviated IT). The first way of learning refers to the improvement of the global climate-observation system. Such improvements increase the precision of the temperature observations and thus reduce estimation errors for the equilibrium climate sensitivity. The second way of learning refers to the reconstruction of the historical temperature records. Such activities have been widely conducted during the last 10-20 years (Brohan et al., 2009) and the reconstructed records can be used to update the probability distribution of the climate sensitivity. The third way of learning is to produce new information other than temperature records and then update the current belief, utilizing the improved theory. For instance, Annan and Hargreaves (2006) and Hegerl et al. (2006) use paleoclimate data to constrain a credible range of the climate

sensitivity. Urban and Keller (2009) use additional data such as radiative forcing along with temperature observations to estimate the distribution of the climate sensitivity.

This paper proceeds as follows. Section 2 discusses in more detail the way we implement active learning. Section 3 describes our climate-economy model with active learning as well as computational methods used. Section 4 illustrates the results. Section 5 concludes.

2 Implementation of active learning

Improved observations (IO)

The rate of learning about the climate sensitivity is highly sensitive to temperature shocks, which are composed of measurement errors, data coverage bias (the number and the distribution of weather stations), model accuracy to match the true state of nature, natural variability, and so on (Brohan et al., 2006; Rayner et al., 2006). As the variance of temperature shocks increases (decreases, respectively), the signal to noise ratio falls (grows, respectively) and thus makes it difficult (easy, respectively) to detect the true state. Webster et al. (2008) and Hwang et al. (2013a), among others, show the sensitivity of the rate of learning to temperature shocks.

Temperature shocks are subject to change as the observational system improves. For instance, as illustrated in Figure 1, observational errors of the global mean surface air temperature have decreased over time as the number of observational instruments such as weather stations, voluntary observational ships, drifting buoys and moored buoys has been increased.

We estimate the global expenditure on temperature observations by multiplying the number of observational instruments with the unit cost of each instrument. See Appendix A for more detail. The total cost of the current global temperature observational system is estimated to

\$450M in 2005,⁶ a ballpark estimate. We investigate the sensitivity of our results to the other cost estimates in Section 4.

Assuming independence among observational errors, model accuracy, and natural variability, the variance of global mean temperature can be decomposed into three elements:

$$\sigma_{\varepsilon}^2 = \sigma_{ob}^2 + \sigma_m^2 + \sigma_{nv}^2 \quad (1a)$$

where σ_{ob} , σ_m , and σ_{nv} denote observational errors, model accuracy to match the true state of nature, and natural variability, respectively.

Natural variability is assumed to be constant over time. This represents the fact that the natural variability is not controlled by the decision maker, and the assumption that climate change does not affect natural variability.

Note that the decision maker does not change the temperature response model in our active learning model, but model accuracy varies as the model is re-estimated.

Broadly speaking, the uncertainty about the global mean air temperature is linearly related to the reciprocal of the number of observational instruments (Jones et al., 1997; Brohan et al., 2006; Rayner et al., 2006). Assuming independence between the sea surface temperature

⁶ For comparison, the United States spent \$140M on *in-situ* climate observations in 2010 (submission of USA to UNFCCC/SBI 35). WMO and UNEP (2010) estimated the global expenditure on climate observations as \$4~6 billion/yr. However their estimate includes all kinds of observations including satellite observations and radiosonde, and their estimate is not split by each component (personal communication with the GCOS secretariat of WMO). Thus we cannot directly compare the estimate with ours. Douglas-Westwood (2006) estimated the total cost of ocean observations as \$402M in 2005 by investigating global funds for ocean observational system. Their estimate also includes all forms of observations and research efforts.

(SST) and the land air temperature (LAT), the variance of the global mean temperature is calculated as follows.

$$\sigma_{ob_t}^2 = \sum_j \omega_j \sigma_{ob_{j,t}}^2 = \sum_j \omega_j (\alpha_j / N_{oj} + \beta_j) = \sum_j \omega_j (c_j \alpha_j / K_{R_1,j} + \beta_j) \quad (1b)$$

where j refers to the land (l) or sea observations (s), ω is the weight for calculating the mean, proportional to the respective area of the sea and the land, N_o is the number of weather stations on land or the number of sea surface temperature observations, K_{R_1} is the capital stock invested in observational system, α and β are coefficients calibrated to the uncertainty about the global mean air temperature and the number of observations, and c is the conversion factor of N_o to K_{R_1} . For simplicity, we assume that as the investment in observational system becomes higher, the observational errors approach zero ($\beta = 0$).

Substituting Equation (1b) into (1a) leads to:

$$\begin{aligned} \sigma_{\varepsilon,t}^2 &= \omega_l [c_l \alpha_l / p K_{R_1,t}] + (1 - \omega_l) [c_s \alpha_s / \{(1 - p) K_{R_1,t}\}] + \sigma_{mo}^2 + \sigma_{nv}^2 \\ &= \alpha_{R_1} / K_{R_1,t} + \beta_{R_1} \end{aligned} \quad (1c)$$

where $\alpha_{R_1} = 2\omega_l c_l \alpha_l + 2(1 - \omega_l) c_s \alpha_s = \$3.42M$ and $\beta_{R_1} = \sigma_{mo}^2 + \sigma_{nv}^2 = 0.0064$.⁷ For the last term of the equation, we assume that the proportion of money spent on land observations (p) is 0.5. Finally, we assume that K_{R_1} should be less than $10^6 \times \alpha_{R_1}$. That is, the decision maker

⁷ β_{R_1} is calculated from the total errors ($\sigma_{\varepsilon,0}^2 = 0.10^2$, following Tol and De Vos (1998)) and the observational errors ($\sigma_{ob,0}^2 = 0.06^2$, estimated from the HadCRUT4 (Morice et al., 2012) dataset).

in our model does not make an effort to reduce the variance of temperature shocks if the difference between β_{R_1} and $\alpha_{R_1}/K_{R_1,t}$ is smaller than 10^{-3} .

Equation (1c) says that the variance of temperature shocks decreases as the investment in temperature observations increases. If there is no investment, the research capital does not increase, and thus the observational errors do not change. The only channel of learning in this case is learning from temperature observations – or passive learning similar to the model of Hwang et al. (2013a).

Additional observations from the past (AO)

Currently, the analysis of the global mean temperature depends on instrumental records after the year 1850 (Smith et al., 2008; Hansen et al., 2010; Morice et al., 2012). However, pre-1850 climate data are also stored in e.g. marine logbooks and they have been actively recovered for the last 10-20 years (Garcia-Herrera et al., 2005; Brohan et al., 2009; Wilkinson et al., 2011). Such efforts increase the number of observations. However, the limited coverage of time and space, and the inconsistency between recovered data make the estimation of the global mean temperature difficult.⁸

Considering these caveats, we assume that there is a minimum number of data for estimating the annual mean of global air temperature. Regarding this, Jones et al. (1997) estimate that 100 or less independent observational sites are required for estimating the annual mean temperature. Assuming each marine logbook contains a month's climate observations

⁸ The recovered data do not usually cover the entire world and thus observations were sporadic. For instance, ships usually followed routes designed for purposes other than climate observations. In addition, weather observations were not required for the ship owners and the measurement technique was not standardized until the early 1850s (Brohan et al., 2009). Therefore the consistency of temperature records between different ships is not guaranteed.

(Brohan et al., 2009), about 1,200 logbooks covering independent observational sites are needed for producing an additional temperature record equivalent to the current global mean air temperature observation. Estimating from the UK Colonial Registers and Royal Navy Logbooks project (Wheeler, 2009) and the CLIWOC project (Garcia-Herrera et al., 2005), recovering data from a logbook costs about \$850. Consequently in our assumption the cost for reconstructing a year's temperature is about \$1M.

However there is no guarantee that the recovered data are independent of each other.

Alternatively, if we assume that the similar number of records to the number of the current observational system should be recovered to estimate the global mean temperature, it costs about \$300M for reconstructing an additional data point.⁹ This estimate is more reasonable than the previous one, and thus we use the unit cost of \$300M for one observation. Again this is a ballpark estimate which will be subjected to sensitivity analysis in Section 4.

Since the number of reconstructed observations (N_h) can be any nonnegative integer, we apply a Poisson distribution for N_h as in Equation (2).

$$\Pr(N_{h,t} = n) = \lambda^n e^{-\lambda} / n! = (R_{2,t} / \alpha_{R_2})^n \exp(-R_{2,t} / \alpha_{R_2}) / n! \quad (2)$$

where R_2 is the amount of money invested in data-recovery projects, $n = 0, 1, 2, \dots$ is the number of reconstructed data points, $\lambda = R_2 / \alpha_{R_2} \geq 0$ is the expected number of reconstructed data, $\alpha_{R_2} (= \$300M)$ is the correction factor for calculating λ from R_2 . We set an upper bound on $N_h \leq 1,000$ because there may be natural limits for the reconstruction of temperature records. Consequently, R_2 has an upper bound: $R_2 \leq 1,000 \times \alpha_{R_2}$.

⁹ We multiply the current number of observational instruments (e.g., land stations and ships) by the unit cost for recovering a logbook.

Equation (2) says that the expected number of additional observations increases in research investment. If there is no investment (in turn no additional observation from the past) the only channel of learning is the yearly (instrumental) observation – or passive learning.

Finally, we assume that the reconstructed historical global mean temperature is also affected by random (normal) shocks with mean zero. Accounting for the fact that the temperature observational system has been improved over time, we set the variance of additional observations from the past at $(0.14)^2$, which is similar to the average of the late 19th century.

Improved theory (IT)

Investment in climate science enhances the understanding of the climate process, resulting in improved theory. For instance, the long history of climate, revealed by palaeoclimate research hundreds of thousand years back, is a good reference for current climatic change (Snyder, 2010). Reconstructed climate data from ice-cores can also be used to construct the distribution of the climate sensitivity (Edwards et al., 2007; Rohling et al., 2012). Beside palaeoclimate research, there are a number of ways to construct a probability distribution of the climate sensitivity, including research on volcanic cooling, radiative forcing (particularly aerosols), observations of heat uptake by the ocean and research into the atmospheres of other planets (Hegerl et al., 2006; Urban and Keller, 2009). For instance, Annan and Hargreaves (2006) take the distribution of the climate sensitivity estimated from the instrumental temperature observations as a prior and then update the prior using the information gathered from climate research. We follow this approach in this paper.

We first investigate the effectiveness of investment in climate science on the accumulation of scientific knowledge. As a measure of scientific knowledge we consider the number of articles published worldwide in geoscience. Figure 2 shows the result. We see that investment in climate research is effective in producing knowledge, or at least journal articles – and that

the relationship is roughly linear. With this evidence, we assume that knowledge on the true value of climate sensitivity, one of the key research outputs in climate science, is proportional to investment in climate research.

In addition, we assume that the decision maker would consult peer-reviewed studies, for example as assessed by the Intergovernmental Panel on Climate Change (IPCC)'s Assessment Reports, in order to use the probability distribution constructed from climate research for updating the prior - the distribution estimated from instrumental temperature records. The peer review process is conducted on a regular basis and each probability distribution of the climate sensitivity obtained from climate research is summarized into a representative distribution. The price of learning is calculated as the historical investment in climate research divided by the historical variance reduction. The global expenditure on climate research from the year 1990 was \$150 billion; see Figure 2. Such expenditure produced an independent distribution of the climate sensitivity such as the one in Rohling et al. (2012): the lognormal distribution with mode $2.9^{\circ}\text{C}/2\text{xCO}_2$.¹⁰ The confidence interval of Rohling et al. (2012) is about 10% wider than the one of the climate sensitivity distribution in the 4th IPCC assessment report (Solomon et al., 2007). Thus for simplicity, we assume that the variance of the total feedback factors constructed from climate research is also 10% higher than 0.13^2 estimated from temperature observations.

In addition, we assume that the mean of the total feedback factors acquired from climate research is randomly drawn from the following distribution.

$$f_{R_{3,t}} \sim N(\overline{f_{R_{3,t}}}, \sigma_{R_{3,t}}^2) \quad (3a)$$

¹⁰ Rohling et al. (2012) summarized the outcomes of 17 independent palaeoclimate research projects from the early 1990s to 2012, and then derived the climate sensitivity distribution.

$$\overline{f_{R_3,t}} \sim N(f, \sigma_0^2) \quad (3b)$$

$$\sigma_{R_3,t}^2 = (\alpha_{R_3}/K_{R_3,t}) \cdot \sigma_{R_3,0}^2 \quad (3c)$$

where ‘ $\sim N(m, var)$ ’ reads as ‘follow the normal distribution with mean m and variance var ’, $\overline{f_{R_3,t}}$ is the total feedback factors estimated from climate research, $\overline{f_{R_3,t}}$ and $\sigma_{R_3}^2$ are the mean and the variance of the distribution of $f_{R_3,t}$, respectively, f and σ_0^2 are the (pre-specified) true value and the initial variance (in 2005) of the total feedback factors, respectively, K_{R_3} is the investment in climate research, α_{R_3} ($=\$33,462M$) is the correction factor for calculating $\sigma_{R_3}^2$ from K_{R_3} ,¹¹ $\sigma_{R_3,0}^2 (=1/12)$ is the variance of the uniform distribution $U(0, 1)$, representing ignorance when there is no investment in climate research. We set a lower bound on $\sigma_{R_3,t}^2 \geq v_t$ because the information from climate research such as palaeoclimate data is not as accurate as the one from instrumental observations. Consequently, K_{R_3} has an upper bound:

$$K_{R_3} \leq \sigma_{R_3,0}^2 / \sigma_{R_3}^2 \times \alpha_{R_3}.$$

Equation (3c) shows that the distribution of the total feedback factors constructed from climate research changes as the scientific knowledge accumulates. If there is no investment, no additional information is available and thus the only way of learning is temperature observations – or passive learning. These formulations ensure the required independence (Annan and Hargreaves, 2006) between instrumental temperature observations and data from climate research.

¹¹ $\alpha_{R_3} = \text{Global expenditure} \times \frac{\text{reduced variance from the expenditure}}{\text{initial variance}} = \$150 \times 10^3 M \times \frac{1.1 \times 0.13^2}{1/12} = \$33,462M$. Together

with Equation (3c), this ensures that if the decision maker spends \$150 Billion on climate research, the variance of the total feedback factors estimated from climate research decreases to 1.1×0.13^2 , which is consistent with the historical observations. If she spends more (respectively, less) the variance reduces more (respectively, less).

3 Model and methods

Climate-economy model

Hwang et al. (2013a) introduce deep uncertainty and Bayesian learning on the climate sensitivity into the DICE model, and use the model to investigate the rate of learning, the optimal level of emissions control, and the value of learning. However, learning in their model is passive in that the decision maker does not make an explicit effort to reduce uncertainty except for using annual temperature observations to revise her belief on the climate sensitivity. Here we revise the model of Hwang et al. (2013a) to introduce active learning. Below, we only present the model as it differs from Hwang et al. (2013a).

The decision maker in our model chooses the rate of emissions control and the level of investment in climate research each time period so as to maximize social welfare defined as in Equation (4) – the expected discounted sum of utility of consumption.¹² The gross outputs net of damage costs and abatement costs are allocated into consumption, investment in general, and investment in climate research. For simplicity, the rate of saving is assumed to be constant.¹³

$$\max_{\mu_t, R_{i,t}} \mathbb{E} \sum_{t=0}^{\infty} L_t \beta^t u(C_t, L_t) \quad (4)$$

$$C_t = (1 - \theta_1 \mu_t^{\theta_2}) \Omega_t Q_t - I_t - R_{i,t} \quad (5)$$

¹² As in Hwang et al. (2013a) we apply the annual time step and a finite difference method into the original DICE model (Nordhaus, 2008). These formulations increase the accuracy of the approximation on the continuous nature of the economy and climate. For a discussion on this issue, see Cai et al. (2012).

¹³ This does not have significant impacts on the results. See Hwang et al. (2013a) on this point.

where \mathbb{E} is the expectation operator given information at point in time t , $u=(C_t/L_t)^{1-\alpha}/(1-\alpha)$ is the instantaneous utility of per capita consumption, C_t is consumption, L_t is labor force, $\mu_t(0\leq\mu_t\leq 1)$ is the emissions control rate, $I_t=s_tQ_t\Omega_t$ is the investment, $s_t(=0.245)$ is the savings' rate (following Hwang et al. (2013a)), $\Omega_t=1/(1+\kappa_1T_{AT_t}+\kappa_2T_{AT_t}^{\kappa_3})$ is the damage function, $Q_t=A_tK_t^\gamma L_t^{1-\gamma}$ is the gross output, $R_{i,t}(0\leq R_{i,t}\leq \overline{R_{i,t}})$ is investment in climate research i ($=1, 2, 3$ denoting each way of active learning, see Section 2), $\overline{R_{i,t}}$ is the upper bound for $R_{i,t}$ (see Section 2), T_{AT_t} is the atmospheric temperature change (w.r.t. 1900), A_t is the total factor productivity, K_t is the capital stock, $\alpha(=2)$ is the elasticity of marginal utility of consumption, $\beta=1/(1+\rho)$ is the discount factor, $\rho(=0.015)$ is the pure rate of time preference, $\gamma(=0.3)$ is the elasticity of output with respect to capital, $\kappa_1(=0)$, $\kappa_2(=0.0028388)$, $\kappa_3(=2)$, $\theta_1(=0.0561)$, and $\theta_2(=2.8)$ are parameters. The parameter values are the same as in DICE2007 unless noted otherwise.

The knowledge stock accumulates as follows:

$$K_{R_i,t+1} = (1 - \delta_{R_i})K_{R_i,t} + R_{i,t} \quad (6)$$

where K_{R_i} is the knowledge stock for each channel of active learning, δ_{R_i} is the depreciation rate of research investment. For simplicity we assume that the knowledge stock does not depreciate ($\delta_{R_1} = \delta_{R_2} = \delta_{R_3} = 0$).

Bayesian learning

The decision maker in our model updates her belief on the climate sensitivity - through updating the belief on the total feedback factors - based on temperature observations and information from climate research, using Bayes' Theorem:

$$p(f|T_{AT}, D) \propto p(T_{AT}, D|f) \times p(f) \propto p(T_{AT}|f) \times p(D|f) \times p(f) \quad (7)$$

where f is the total feedback factor, normally distributed with mean \bar{f}_t (the initial value is 0.60 of which corresponding climate sensitivity is 3°C), and variance v_t (the initial value is 0.13² following Roe and Baker (2007)), $p(f)$ is the prior distribution of the total feedback factors, $p(T_{AT}, D|f)$ is the joint likelihood function of the observations (T_{AT}) and data (D) from climate research, $p(T_{AT}|f)$ and $p(D|f)$ are the likelihood functions of the observations and data, respectively, and $p(f|T_{AT}, D)$ is the posterior distribution. For the last term in the equation, we assume that the instrumental temperature records and the information from climate research are mutually independent.

Throughout the models of active learning, we use the normal distribution of Roe and Baker (2007) as the initial prior. The likelihood function of each channel of active learning is different from each other according to the respective learning mechanism, and this affects the resulting posterior distribution.

We apply a general technique for Bayesian updating as discussed in Cyert and DeGroot (1974) to derive the posterior distribution. The resulting posterior mean and the variance of the total feedback factors for IO are given in Equations (8a) and (9a). Similarly Equations (8b) and (9b) are for AO and Equations (8c) and (9c) are for IT. For the derivation of these equations in detail, see Appendix B.

$$\bar{f}_{t+1} = \frac{\bar{f}_t + \zeta_1 T_{AT_t} H_{t+1} v_t / v_{\varepsilon,t}}{1 + \zeta_1^2 T_{AT_t}^2 v_t / v_{\varepsilon,t}} \quad (8a)$$

$$\bar{f}_{t+1} = \frac{\bar{f}_t + \sum_{j=1}^{N_{h,t}} \zeta_1 T_{AT_t} H_{t+1,j} v_t / v_{\varepsilon,j}}{1 + \sum_{j=1}^{N_{h,t}} \zeta_1^2 T_{AT_t}^2 v_t / v_{\varepsilon,j}} \quad (8b)$$

$$\overline{f_{t+1}} = \frac{\frac{\overline{f_t} v_{R_{3,t}} + \overline{f_{R_{3,t}}} v_t}{v_{R_{3,t}} + v_t} + \zeta_1 T_{AT_t} H_{t+1} \left(\frac{v_{R_{3,t}} v_t}{v_{R_{3,t}} + v_t} \right) / v_\varepsilon}{1 + \zeta_1^2 T_{AT_t}^2 \left(\frac{v_{R_{3,t}} v_t}{v_{R_{3,t}} + v_t} \right) / v_\varepsilon} \quad (8c)$$

$$v_{t+1} = \frac{v_t}{1 + \zeta_1^2 T_{AT_t}^2 v_t / v_{\varepsilon,t}} \quad (9a)$$

$$v_{t+1} = \frac{v_t}{1 + \sum_{j=1}^{N_{h,t}} \zeta_1^2 T_{AT_t}^2 v_t / v_{\varepsilon,j}} \quad (9b)$$

$$v_{t+1} = \frac{\left(\frac{v_{R_{3,t}} v_t}{v_{R_{3,t}} + v_t} \right) / v_\varepsilon}{1 + \zeta_1^2 T_{AT_t}^2 \left(\frac{v_{R_{3,t}} v_t}{v_{R_{3,t}} + v_t} \right) / v_\varepsilon} \quad (9c)$$

where N_h is the number of additional temperature observations for AO (see Equation (2)), with $N_h = 1$ for IO and IT, $v_{\varepsilon,t}$ evolves over time as in Equation (1) for IO and is fixed at an initial value $v_{\varepsilon,t} = v_\varepsilon$ for AO and IT, $v_{R_{3,t}}$ and $\overline{f_{R_{3,t}}}$ are the variance and the mean of the total feedback factors estimated from climate research for IT as in Equation (3) and are ignored for IO and AO, $H_{t+1,j} \equiv T_{AT_{t+1},j} - \zeta_2 T_{AT_t} - \zeta_3 \ln(M_t/M_b) - \zeta_4 T_{LO_t} - \zeta_5 R F_t$, where j refers to the j th random observation from the past (see Section 2).

The posterior distribution calculated from Equations (8) and (9) serves as the prior for the next time period. In this way the decision maker learns about the true value of the total feedback factors each time period.¹⁴

¹⁴ That is, the decision maker in our model can be considered as a Bayesian statistician who minimizes the value of risk – or the expected value of the loss function - in an estimation problem. If we assume a quadratic loss function, the Bayes decision minimizing the value of risk is the mean of the posterior distribution. Furthermore, the corresponding risk is the variance of the posterior distribution. See DeGroot (1970) for more details.

Finally, uncertainty about the climate sensitivity is derived from the distribution of the total feedback factors through Equation (10) (Roe and Baker, 2007).

$$\Delta T_{2 \times CO_2} = \Delta T_{2 \times CO_2,0} / (1 - f) \quad (10)$$

where $\Delta T_{2 \times CO_2}$ is the equilibrium climate sensitivity and $\Delta T_{2 \times CO_2,0}$ ($=1.2^\circ\text{C}/2\times\text{CO}_2$, following Roe and Baker (2007)) is the reference climate sensitivity in a grey-body planet. f is assumed to be strictly less than 1. Specifically we set the upper bound of f at 0.99999 for our simulations, of which corresponding value of the climate sensitivity is about $100,000^\circ\text{C}/2\times\text{CO}_2$.

Computational methods

In order to solve the active learning model, we reformulate the problem in a recursive way as in Equation (11) (a Bellman equation), and then solve the model over an infinite time horizon with a dynamic programming method proposed by Maliar and Maliar (2005) and generalized by Judd et al. (2011).

$$W(\mathbf{s}_t, \boldsymbol{\theta}_t) = \max_{\mathbf{c}_t} [u(\mathbf{s}_t, \mathbf{c}_t, \boldsymbol{\theta}_t) + \beta \mathbb{E}_t W(\mathbf{s}_{t+1}, \boldsymbol{\theta}_{t+1})] \quad (11)$$

$$W(\mathbf{s}_t, \boldsymbol{\theta}_t) \approx \sum_{n=1}^N \psi_i(\mathbf{s}_t, \boldsymbol{\theta}_t; \mathbf{b}_n) \quad (12)$$

where $W(\mathbf{s}_t, \boldsymbol{\theta}_t)$ is the value function starting from period t , \mathbf{c} is the vector of control variables (μ, R_i) , \mathbf{s} is the vector of state variables $(K, K_{R_i}, M_{AT}, M_U, M_L, T_{AT}, T_{LO}, \bar{f}, v, L, A, \sigma)$, M_U and M_L are the carbon stocks in the upper and lower ocean, respectively, σ is the emissions-output ratio, $\boldsymbol{\theta}$ is the vector of uncertain variables (f, ε, N) , ψ is the basis function,

\mathbf{b} is the vector of coefficients for the basis function, and $n(=1, 2, \dots, N)$ refers to each element – state variables and uncertain variables – of the approximated basis function.

The solution algorithm is similar to the one of Hwang et al. (2013a) and it can be summarized as follows. For the solution algorithm in detail and the accuracy of the method see Hwang et al. (2013a). First, approximate the value function with a flexible basis function such as logarithmic function. Second, find the first order conditions (see Appendix C). Third, choose an initial guess on coefficients $\mathbf{b}^{(0)}$ of the basis function. Fourth, simulate a time series of variables satisfying the first order conditions, transitional equations, and boundary conditions with the initial guess. Fifth, calculate the left hand side and the right hand side of the Equation (11) using the simulated time series, and then find $\hat{\mathbf{b}}$ that minimizes the difference between them.¹⁵ Sixth, update the initial guess on $\mathbf{b}^{(0)}$ using a pre-specified updating rule. Seventh, iterate the above process with the new guess until the value function converges.¹⁶

Since the model is highly nonlinear and the control variables are bounded, we numerically solve the model using a Newton method with Fisher's function. See Judd (1998) and Miranda and Fackler (2004) for the method in detail.

Most literature solving an integrated assessment model of climate change with *infinite* time horizon take a similar approach to ours. They formulate the problem in a recursive way, approximate the value function with a flexible basis function, and use the fixed-point theorem (see Sockey and Lucas, 1988) to find solutions. The main difference between literature is the approximation methods they apply as follows: neural networks (Kelly and Kolstad, 1999;

¹⁵ In order to find $\hat{\mathbf{b}}$ we applied the least square - singular value decomposition (LS-SVD) method. We used the Gauss-Hermite quadrature method for conditional expectation. For the methods in detail see Judd et al. (2011).

¹⁶ The tolerance level for convergence was set to 10^{-4} .

Leach, 2007), spline approximation (Kelly and Tan, 2012), Chebyshev polynomials (Traeger, 2012), and logarithmic approximation (Hwang et al., 2013a). In addition, Hwang et al. (2013a) search for solutions on an ergodic set (i.e., on a simulated time series satisfying the first order conditions, see Judd et al. (2011) for this issue), whereas the others search for solutions on a carefully designed grid. The method of Hwang et al. (2013a), which the current paper also applies, is less prone to the ‘curse of dimensionality’ than the others. For instance since our active learning model has 10 state variables (if we include the time variable as an argument for the value function approximation instead of exogenous variables), the number of the total grid points will be 10^{10} , which is far greater than computational constraints, if we try to apply the method of the other literature with 10 grid points per each variable. The other difference of Hwang et al. (2013a) from the literature is that they include (time dependent) exogenous variables as arguments for the value function in order to account for the time dependence of the model, whereas the others include the time variable as an argument for the value function. Regarding the time dependence, Cai et al. (2012b) who solve a *finite* time horizon problem let the coefficient of the basis function vary every time period.

Accounting for random realizations of uncertain variables, we do 1,000 Monte Carlo simulations for each channel of active learning and present the average of all simulations in Section 4.

4 Active learning and climate policy

Research investment

As illustrated in the top left panel in Figure 3, the optimal level of investment in climate research is much higher than the current level of annual expenditure. For instance, the optimal level of investment in observational systems (IO), the reconstruction of historical

temperature records (AO), and climate research in general (IT) are about \$340 billion/yr (as opposed to the current level of \$450 million/yr), \$83 billion/yr (as opposed to \$300 million/yr), and \$4 trillion/yr (as opposed to \$75 billion/yr), respectively, in 2005. These results confirm that the benefits of learning are far greater than the costs of learning (Keller et al., 2007; Baehr et al., 2008; Pindyck, 2012). We also observe that the initial level of research investment is far greater than that in the near future. Especially for the IO case, there is no significant investment after the initial peak. For the other cases, at least for the 21st century, the initial level of investment is the greatest – although this is not clear from the figure.¹⁷ This reflects the point that early investment in reducing uncertainty is beneficial because (1) early investment benefits from a longer future and (2) knowledge saturates in our model specification. After the initial peak the research capital stock gradually increases for a certain period of time until knowledge saturates.¹⁸ This is because the rate of variance-reductions from climate research ($\partial v_{t+1}/\partial K_{R,t}$) diminishes as research capital accumulates (see Equation 9). Put differently, after the initial peak in research investment, far more efforts are required for further variance-reductions, one unit of variance-reduction becoming more expensive over time.

The variances of the temperature shocks (IO) and of the additional information obtained from climate research (IT) fall rapidly. The additional number of temperature observations (AO) behaves as the research capital stock does.

The rate of learning is sensitive to the cost estimates for each channel of active learning, of course, as the costs of learning increase the rate of learning decreases. For an illustration, we

¹⁷ For AO, after the 22nd century, the level becomes greater than the level in 2005.

¹⁸ Note that we set bounds on research investment for reasons of physical constraints (see Section 2), and thus the saturation of knowledge refers to the point where the bounds start to bind.

present in Figure 3 the case where the unit costs of learning are increased tenfold. As the cost of learning increases, the optimal level of investment decreases since the constraints bind (see Section 2) at different times (see the top left panel). In addition the rate of learning decreases as the unit cost of learning increases (see the top right, bottom left, and bottom right panel).

The reduction of uncertainty

Figure 4 illustrates the evolution of the parameter values of the climate sensitivity distribution. For comparison, we also present the values of passive learning where only learning from temperature observations is present. The mean parameter \bar{f} converges to the pre-specified true value although the rate of convergence differs from case to case. The variance parameter v approaches – but never reaches – zero over time. The rate of learning is higher under active learning than under passive learning. This is by construction, as each form of active learning constitutes an additional way of having information on the true value. For all the cases, the probability density in the tail of the climate sensitivity distribution shrinks over time.

The optimal carbon tax and income

We compare the optimal carbon tax and income for each form of active learning. Figure 5 shows the results. First of all, as expected, the optimal carbon tax is highest in the uncertainty case and is lowest in the deterministic case.¹⁹ Passive learning lowers the carbon tax and active learning strengthens this further. Numerically, for instance, the optimal carbon tax in 2015 of PL, IO, AO, and IT case is reduced by 2.8%, 5.1%, 9.5%, and 10.5% from the carbon tax of the uncertainty case (\$34.4/tC in 2015), respectively.

¹⁹ The carbon tax is calculated as a Pigovian tax as in the original DICE model. $tax = -\alpha(\partial W/\partial E_t)/(\partial W/\partial K_t)$, where E_t and K_t are greenhouse gas emissions and the capital stock, respectively, and α is a constant.

The difference in income - the sum of consumption and investment, excluding research investment is not discernible among the cases. For further results see Appendix D.

5 Conclusion

In this paper we implemented active learning on climate change into a climate-economy model. Specifically we introduced three ways of active learning into the DICE model, namely improved observations, additional observations, and improved theory. We found that the decision maker – a Bayesian statistician – tries to reduce the uncertainty on climate change through investment in climate research. That is, the decision maker invests a significant amount of money, far more than the current level of expenditure, in climate research in order to increase the rate of learning.²⁰ This helps to increase the possibility of the decision maker to take actions contingent on the true state of the world, i.e. take improved decisions. Indeed the level of uncertainty measured as the variance parameter of the climate sensitivity distribution shrinks more rapidly in the active learning case than in the passive learning case. As such, the optimal level of carbon tax for the active learning model is nontrivially lower than for the case where the decision maker does not have an option to learn (the uncertainty model) or where she simply gets information from temperature observations (the passive learning model).

This is one of the first attempts to introduce active learning into an integrated assessment model of climate change. We employed a conventional uncertainty representation and model specifications in this paper. Applying other specifications such as more reactive damage function (Weitzman, 2012), an alternative temperature response model (Hwang et al., 2013b), alternative utility representation (Sterner and Persson, 2008; Millner, 2013; Hwang et al.,

²⁰ This is consistent with earlier estimates of the value of information (e.g., Nordhaus and Popp, 1997).

2013b), and so on would help understand the role of active learning. However a general conclusion would be that as the effect of uncertainty grows, learning plays a more significant role in reducing the tail effect. Regarding numerical analysis, our solution method can be applied to such applications, although resource constraints such as computation time may become binding as the number of variables increases.²¹

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²¹ The solution time for the passive learning model was about 2 to 3 hours, but the active learning model took about 20 to 40 hours according to the specifications of the model on a laptop with 8GB RAM and Intel CORE i5 processor.

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Appendix A. Data for the global climate observational system

The global mean land air temperature is estimated from the records of each country's weather stations. The global mean sea surface temperature is estimated from reports of observational platforms such as ships, drifting buoys, and moored buoys. The number of each system is drawn from the following sources, respectively – number in 2005. Land weather stations (Kennedy et al., 2011a): 3,455 – stations used for calculating CRUTEM4, voluntary observing ships (VOSs) (<http://www.bom.gov.au/jcomm/vos>): 5,429, drifting and moored buoys (<http://www.aoml.noaa.gov/phod/dac>): 1,267 and 194.

The costs for each system are as follows. Land weather station: \$40K per installation, \$60K for annual maintenance on average (Mburu, 2006), VOS: \$4K~\$55K per instrumentation of the VOS package and AWS (Kent et al., 2010), Drifting buoy: \$4.5K~\$7.8K per deployment and annual maintenance (Meldrum et al., 2009), Moored buoy: \$1.15M~\$2.7M per deployment – operational life is 10 years, \$200K~\$500K for annual maintenance (Detrick et al., 2000), Transmission cost – satellite communication system: \$0.12~\$0.64 per observation (North, 2009)

Appendix B. Bayesian updating

To derive the posterior distribution we apply Bayes Theorem. As illustrated in Section 3, we assume that the prior $p(f)$ at time t has a normal distribution with mean \bar{f}_t and variance v_t . The likelihood function of data from climate research $p(D|f)$ also has a normal distribution with mean $\bar{f}_{R_3,t}$ and variance $v_{R_3,t}$, assuming a uniform prior for the data from climate research. The likelihood function of temperature observations $p(T_{AT_{t+1}}|f)$ is normal with mean $(\zeta_1 f + \zeta_2)T_{AT_t} + \zeta_3 \ln(M_t/M_b) + \zeta_4 T_{LO_t} + \zeta_5 RF_t$ and variance $v_{\varepsilon,t}$, since the temperature shocks have normal distribution with mean 0 and variance $v_{\varepsilon,t}$. With the number of additional temperature observations being higher than 1 ($N_{h,t} > 1$, for AO), $p(T_{AT}|f)$ is the product of individual likelihood functions, assuming independence between data. As illustrated in AO, for simplicity, we assume that reconstructed temperature from AO has the same distribution with that of the model year t . Following the assumption on each channel of active learning (see Section 2), $p(D|f)$ is a uniform distribution for IO and AO.

With the information above the posterior distribution is calculated as follows.

$$\begin{aligned}
 & p(f|T_{AT_{t+1}}, D) \\
 & \propto \prod_{j=1}^{N_{h,t+1}} \frac{1}{\sqrt{2\pi v_{\varepsilon,t,j}}} \exp \left\{ -\frac{1}{2} \frac{\left[T_{AT_{t+1},j} - ((\zeta_1 f + \zeta_2)T_{AT_t} + \zeta_3 \ln(M_t/M_b) + \zeta_4 T_{LO_t} + \zeta_5 RF_t) \right]^2}{v_{\varepsilon,t,j}} \right\} \\
 & \times \frac{1}{\sqrt{2\pi v_{R_3,t}}} \exp \left\{ -\frac{1}{2} \frac{[f - \bar{f}_{R_3,t}]^2}{v_{R_3,t}} \right\} \times \frac{1}{\sqrt{2\pi v_t}} \exp \left\{ -\frac{1}{2} \frac{[f - \bar{f}_t]^2}{v_t} \right\}
 \end{aligned} \tag{B1}$$

where M_t is the carbon stock in the atmosphere, T_{AT_t} is the atmospheric temperature deviation (from 1900), T_{LO_t} is the lower ocean temperature deviations (from 1900), \bar{f}_t and v_t are the parameters of the climate-sensitivity distribution and \bar{f}_t and is the mean of the total feedback

factors – the initial value of \bar{f}_t varies according to scenarios and the initial value v_t is 0.13² following Roe and Baker (2007), v_ε is the variance of the stochastic temperature shocks (=0.1² following Tol and de Vos (1998)), RF_t is the radiative forcing from non-CO₂ gases (exogenous), E_{LAND_t} is carbon emissions from the sources other than energy consumption (exogenous), $\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5$, are parameters, M_b is the pre-industrial carbon stock in the atmosphere (=596.4GtC). Other notations are the same as in Section 2 and 3.

Focusing on the posterior kernel, Equation (B1) reduces to Equation (B2). That is, all terms not directly related to f are included in a normalizing constant – not shown for simplicity.

$$\begin{aligned}
p(f|T_{AT_{t+1}}, D) &\propto \exp \left\{ -\frac{1}{2} \sum_{j=1}^{N_{h,t}} \frac{\zeta_1^2 T_{AT_t}^2 f^2 - 2\zeta_1 T_{AT_t} H_{t+1,j} f}{v_{\varepsilon,t,j}} - \frac{1}{2} \frac{f^2 - 2\bar{f}_{R_3,t} f}{v_{R_3,t}} - \frac{1}{2} \frac{f^2 - 2\bar{f}_t f}{v_t} \right\} \\
&\propto \exp \left\{ -\frac{1}{2} \left[\left(\sum_{j=1}^{N_{h,t}} \frac{\zeta_1^2 T_{AT_t}^2}{v_{\varepsilon,t,j}} + \frac{1}{v_{R_3,t}} + \frac{1}{v_t} \right) f^2 - 2 \left(\sum_{j=1}^{N_{h,t}} \frac{\zeta_1 T_{AT_t} H_{t+1,j}}{v_{\varepsilon,t,j}} + \frac{\bar{f}_{R_3,t}}{v_{R_3,t}} + \frac{\bar{f}_t}{v_t} \right) f \right] \right\} \\
&\propto \exp \left\{ -\frac{1}{2} \left[f - \frac{\sum_{j=1}^{N_{h,t}} \frac{\zeta_1 T_{AT_t} H_{t+1,j}}{v_{\varepsilon,t,j}} + \frac{\bar{f}_{R_3,t}}{v_{R_3,t}} + \frac{\bar{f}_t}{v_t}}{\sum_{j=1}^{N_{h,t}} \frac{\zeta_1^2 T_{AT_t}^2}{v_{\varepsilon,t,j}} + \frac{1}{v_{R_3,t}} + \frac{1}{v_t}} \right]^2 \middle/ \left[\frac{1}{\sum_{j=1}^{N_{h,t}} \frac{\zeta_1^2 T_{AT_t}^2}{v_{\varepsilon,t,j}} + \frac{1}{v_{R_3,t}} + \frac{1}{v_t}} \right] \right\}
\end{aligned} \tag{B2}$$

where $H_{t+1,j} \equiv T_{AT_{t+1,j}} - \zeta_2 T_{AT_t} - \zeta_3 \ln(M_t/M_b) - \zeta_4 T_{LO_t} - \zeta_5 RF_t$. $p(D|f)$ is a uniform distribution for IO and AO and $N_{h,t} = 1$ for IO and IT.

Equation (B2) shows that the posterior distribution of the total feedback factors is normal with mean \bar{f}_{t+1} and variance v_{t+1} . Consequently, the posterior mean and the variance become Equations (8a) and (9a) in Section 3 for IO. Similarly, Equations (8b) and (9b) are for AO and Equations (8c) and (9c) are for IT in Section 3.

Appendix C. The first order conditions

We approximate the value function as follows.

$$\begin{aligned}
W(s_t, \theta_t) = & b_0 + b_1 \ln(K_t(f, \varepsilon, N_h)) + b_2 \ln(K_{Ri,t}(f, \varepsilon, N_h)) + b_3 \ln(M_{AT_t}(f, \varepsilon, N_h)) \\
& + b_4 \ln(M_{U_t}(f, \varepsilon, N_h)) + b_5 \ln(M_{L_t}(f, \varepsilon, N_h)) \\
& + b_6 \ln(T_{AT_t}(f, \varepsilon, N_h)) + b_7 \ln(T_{LO_t}(f, \varepsilon, N_h)) \\
& + b_8 \ln(\bar{f}_t(f, \varepsilon, N_h)) + b_9 \ln(v_t(f, \varepsilon, N_h)) + b_{10} \ln(L_t) \\
& + b_{11} \ln(A_t) + b_{12} \ln(\sigma_t)
\end{aligned} \tag{C1}$$

where the notations are the same as in Section 2 and 3.

The first order conditions for optimality are Equation (C2) and (C3).

$$-Q_t \Omega_t \left(\theta_1 \theta_2 \mu_t^{\theta_2 - 1} \right) (C_t / L_t)^{-\alpha} - \beta \frac{\sigma_t Q_t b_3}{(1 - \mu_t) \sigma_t Q_t + E_{LAND_t} + \delta_{aa} M_{AT_t} + \delta_{ua} M_{U_t}} = 0 \tag{C2}$$

$$-(C_t / L_t)^{-\alpha} + \beta \frac{b_2}{(1 - \delta_{Ri}) K_{Ri,t} + R_{i,t}} = 0 \tag{C3}$$

where the notations are the same as in Section 2 and 3.

Appendix D. The solutions of the improved observation case

Applying the methods illustrated in Section 2 and Appendix B and C, we find the solution for each active learning model. We present the solution of the IO model, for illustration.

Although the numbers are different, the qualitative behavior of the variables over time of the other models is similar to Figure D1.

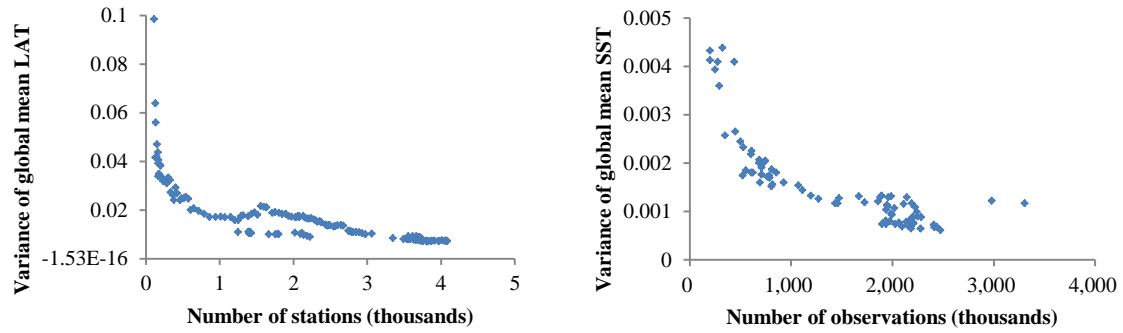


Figure 1 **Uncertainty about global mean temperature. (Left panel):** The variance of global mean land air temperature (LAT) 1850-2006 (CRUTEM3, Brohan et al. (2006)) as a function of the number of weather stations used to estimate the global mean temperature. **(Right panel):** The variance of global mean sea surface temperature (SST) 1925-2006 (HadSST3, Kennedy et al. (2011)) as a function of the number of observations used to estimate the global mean temperature. We obtained the data from John Kennedy.

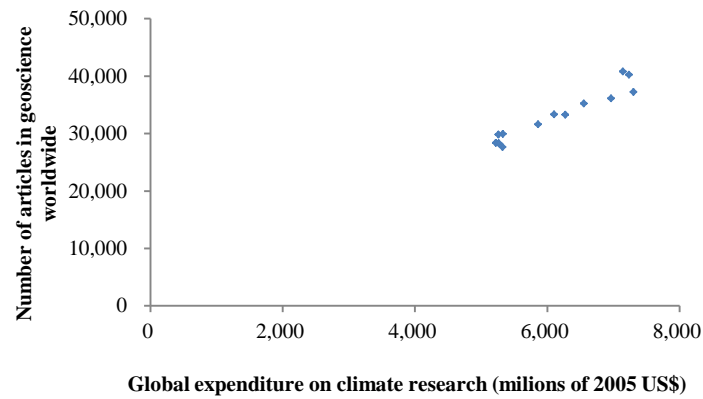


Figure 2 **The relation between the number of articles and global expenditure in climate science field (1995-2007)**. The number of articles is drawn from the U.S. National Science Foundation's *Science and Engineering Indicators 2012*. Global expenditure is drawn from a database (total GBAORD by NABS 1992 socio-economic objectives) of the Statistical Office of European Union (EuroStat) (<http://www.eea.europa.eu/data-and-maps/data/external/eurostat-total-gbaord-by-nabs>).

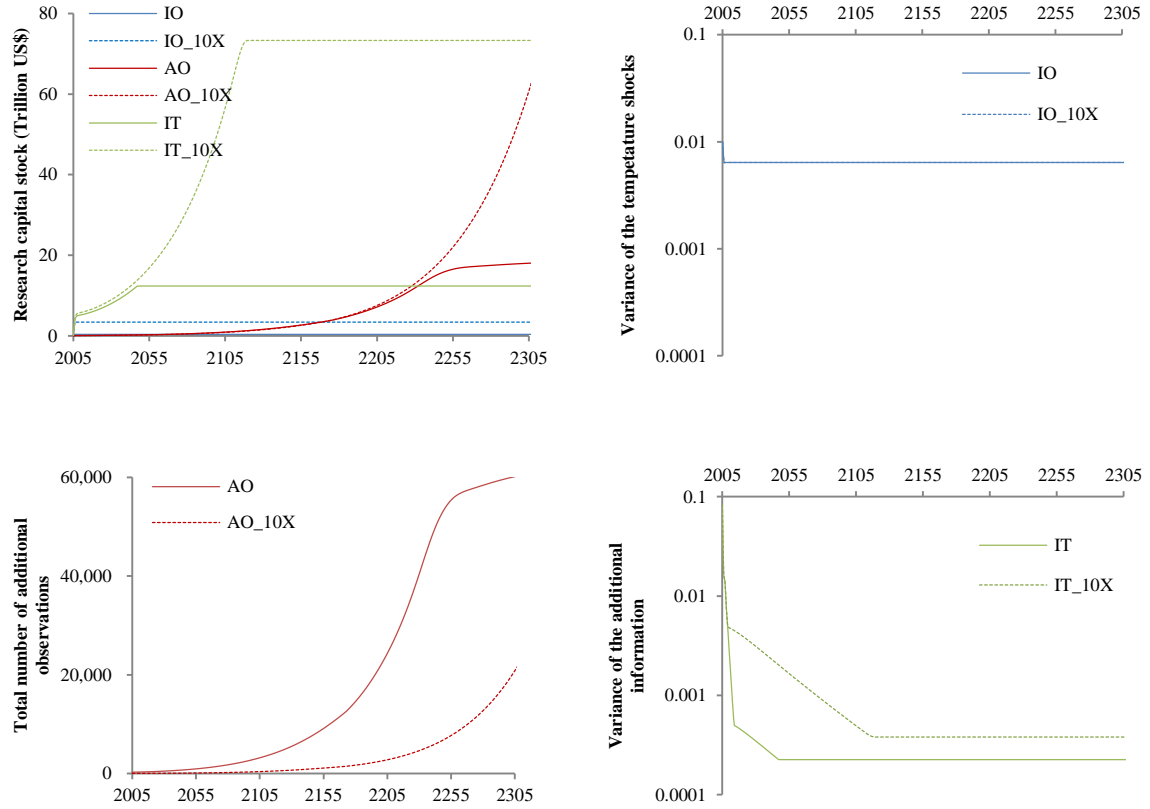


Figure 3 Research investment and the new information gathered. (Upper left panel): The optimal level of research investment. **(Upper right panel):** The variance of temperature shocks for IO. Note that there is no significant difference between IO and IO_10X since the constraint (see Section 2) binds shortly. **(Lower left panel):** The total number of additional observations for AO. **(Lower right panel):** The variance of new information for IT. Note that y-axis is presented on a log scale in each right panel. IO, AO, and IT refer to improved observation, additional observation, and improved theory (see Section 2). 10X stands for cost estimates increased tenfold.

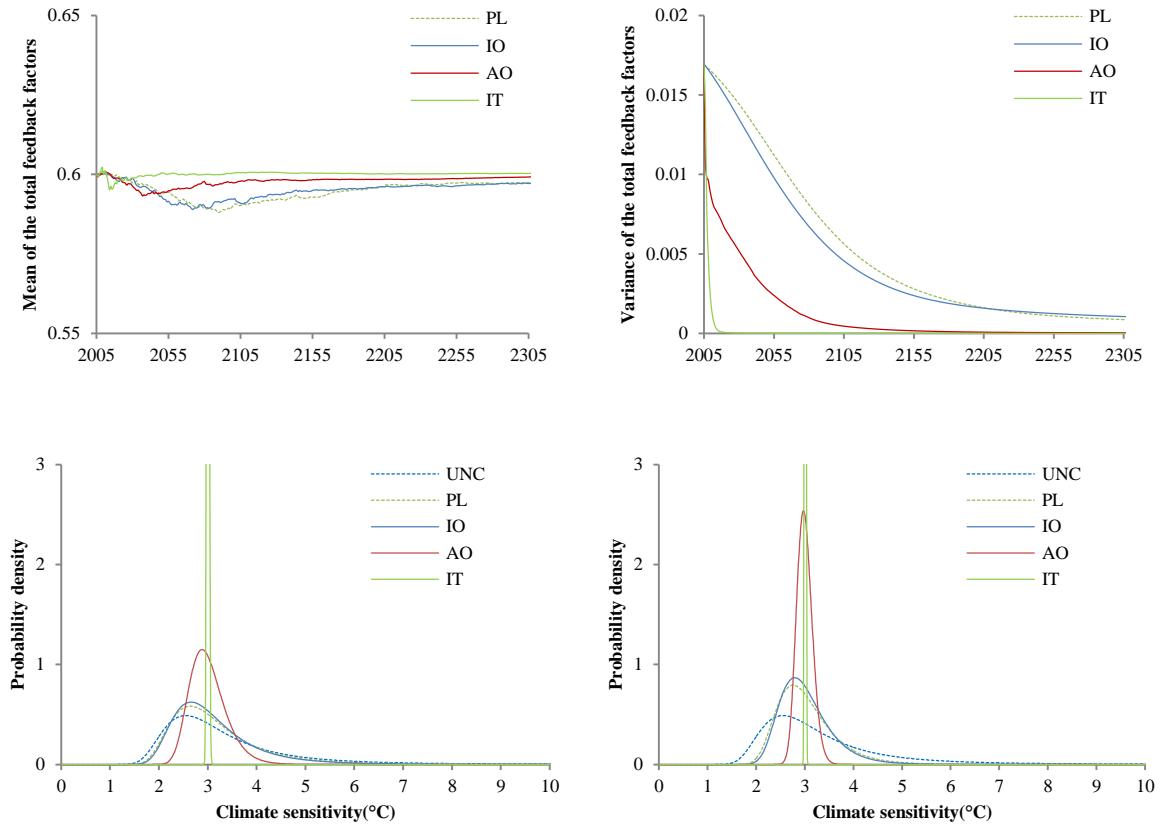


Figure 4 The reduction of uncertainty. (Top left panel): The mean of the total feedback factors. **(Top right panel):** The variance of the total feedback factors. **(Bottom left panel):** Climate sensitivity distribution in 2055. **(Bottom right panel):** Climate sensitivity distribution in 2105. UNC and PL refer to uncertainty (or no learning) and passive learning, respectively. Note that the PDFs of IT are truncated for the purpose of comparison in this figure.

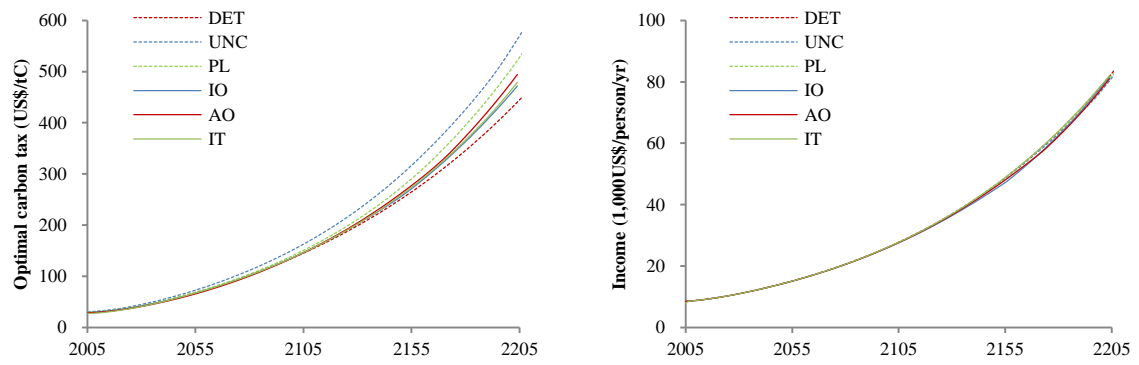


Figure 5 The optimal carbon tax and income. (Left panel): The optimal carbon tax. **(Right panel):** Income. DET refers to the deterministic case.

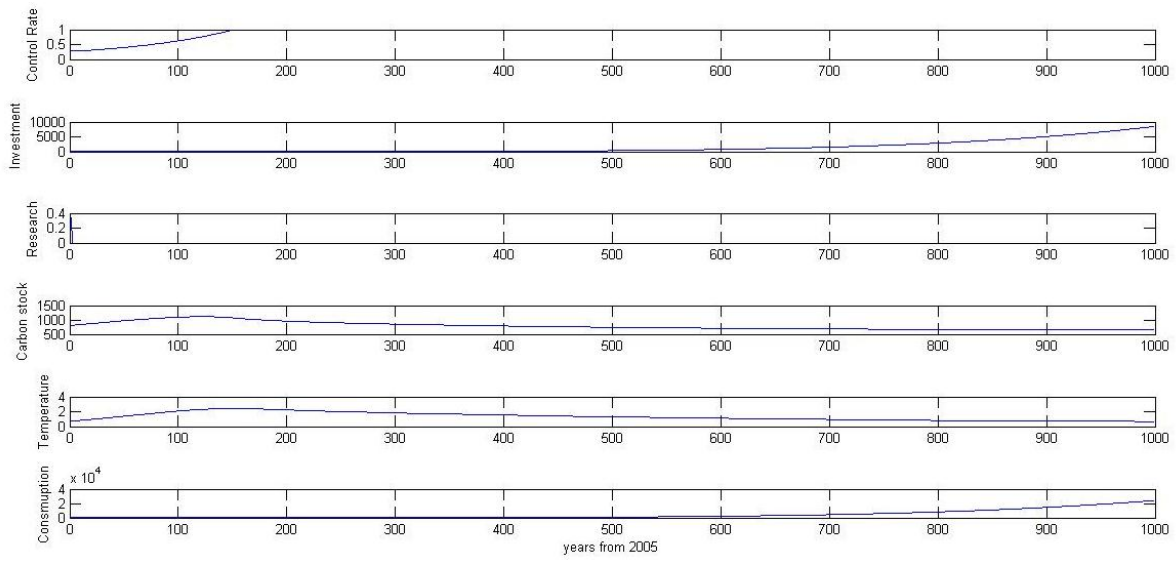


Figure D1 The solutions of the IO model. The unit for investment, research investment, carbon stock, temperature, and consumption are 1,000US\$/person, trillion US\$, MtC, °C, and 1,000US\$/person, respectively.